Variorum - Multi- Disciplinary e-Research Journal
Vol.-01, Issue-III, February 2011

"Applications of Linear Programming Problems and Non Linear Programming Problems in Industry"

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Abstract: The growing complexity and volatility of the business environment has made decision making very difficult. Decision makers can no longer afford to make decisions that are based solely on their experience and observation. Decisions need to be based on data that show relationships, indicate trends and show rates of change in the relevant variables. Quantitative methods help managers to tackle the intricate and complex problems of business and industry, also provide an analytical and objective approach to decision making..Quantitative Methods can be considered as **'statistical'** and **'programming techniques'. Linear programming, game theory, simulation, network analysis, queuing theory** are some of the programming techniques that develop mathematical models which relate the relevant variables in a situation to the outcome, and provide solution to problems in terms of the values of the variables involved. In industry programming techniques can be applied in the areas such as a) Production b) Marketing c) Personnel and d) Finance. In this paper Applications of LPP and NLPP in various areas as well as some case studies are discussed upto formulation stage.

1 Introduction:

1.1 Linear Programming: It is a method of selecting an appropriate optimum combination of factors from a series of alternative which are interrelated and each subject to some constraints or restrictions .It involves the development of linear equation to obtain the best solution for the allocation problem. An allocation problem arise whenever there are number of activities to perform, but limitation an either the amount of resources or the way they can be spent prevent us from performing each separate activity in the most effective way conceivable. In such situation we wish to allot the available resource to the activities in a way that they will optimize the total effectiveness.

> "Linear Programming is a mathematical technique for determining the optimum allocation of resources and obtaining a particular objective when there are alternative uses of the resources : money, manpower, material, machine and other facilities. The objective in resource allocation may be cost minimization or inversely profit maximization. The technique of

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linear programming is applicable to problems in which the total effectiveness can be expressed as a linear function of individual allocations and the limitations on resources give rise to linear equalities or inequalities of the individual allocations"

1.2 Why the name linear programming?

The method of maximizing (or minimizing) a linear function of several variables (**called objective function**) subject to the condition that the variables are non-negative and satisfy a set of linear equations and/or in equations (**called Linear Constraints**) is given the name LINEAR PROGRAMMING.

The term linear implies that all the mathematical relations used in the problem are linear relations, while the term programming refers to the method of determining a particular programme or plan of action. The two together have the technical meaning stated above.

2. Algorithm and Requirements

2.1 Three Steps in Linear Programming:

1. Identify the objective function as a linear function of its variables and state all the limitations on resources as linear equations and/or in equations (constraints).
2. Use mathematical techniques to find all possible sets of values of the variables (unknowns), satisfying the constraints.
3. Select the particular set of values of the variables obtained in (2) that lead to our objective – maximum profit, least cost, etc.

2.2 Basic Requirements

Regardless of the way one defines linear programming, certain basic requirement are necessary before this technique can be employed to optimization problems. These are:

1. **Decision variables and their relationship:** The decision variable refers to any activity (product, project etc.) that is competing with other activities for limited resources. The relationship among these variables should be linear.
2. **Well defined objective function:**. A clearly defined objective must be stated which may be either to maximize contribution by utilizing the available resources, or it may be to produce at the lowest possible cost by using a limited amount of productive factors.
3. **Presence of constraints or restrictions:** There must be limitations on resources (like production, capacity, manpower, time, machines, markets etc.) which are to be allocated among various competing activities.
4. **Quantitative measurement of problem element:**. It is essential that each element of the problem is capable of being quantified. Numerical data must depict the problem in terms

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of relationship involved as well as among the elements considered. Thus, accurate means of measurement, such as rupees, acres, hours, kilogram must be brought into computation.

5. **Alternative courses of action**: There must be alternative courses of action to choose from, e.g., it must be possible to make a selection between various combinations of the productive factors such as men, machines, materials, markets.etc.
6. **Non-negative restrictions:** All decision variables must assume non-negative values as negative value of physical quantities is an impossible situation.
7. **Linearity:** The basic requirements of a linear programming problem are that both the objective and constraints must be expressed in terms of linear equations or inequalities. It is well known that if the number of machines in a plant is increased, the production in the plant also proportionately increases. Such a relationship, giving corresponding increment in one variable for every increment in the other, is called linear and can be graphically represented in the form of a straight line.

3. Applications in Industrial Area

3.1 Industrial Applications

In this area the general objective is to determine a plan for production and procurement in the time period under consideration. It is necessary to satisfy all demand requirements without violating any of the constraints. Some examples of industrial applications are as follows.

3.1.1 Product Mix Problem: An industrial concern has available to itself a certain productive capacity of its various manufacturing process and has the opportunity to utilize this capacity to manufacture various products. Generally different products will have different costs as well as selling prices and therefore will have different unit profits. The problem is the selection of the optimal product mix to make best use of machine and man hours available while maximizing the firm's profit.

3.1.2 Production Scheduling: The situations where several products can be made on each of several different machines; the problem is to decide on a programme which will maximize output, minimize cost or produce some other criterion of efficiency. Linear Programming is useful for allocating operators to machines, products to machines, clerks to particular tasks and in providing optimal connection between production centres and wholesalers.

3.1.3 Production Smoothing Problem: An industrial concern can solve the problem of scheduling its production or procurement over a number of future time periods with the total time span being considered the planning horizon with the help of linear programming approach.

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3.1.4 Blending Problems: When a product can be made from a variety of available raw materials of various composition and prices. The manufacturing process involves mixing some of these materials in varying quantities to make a product, conforming to given specifications serve as constraints in obtaining the minimum cost material blend. These types of problems occur frequently in the petroleum industry, chemical industry, and food industry. In most of these applications, management may decide how much of each resource to purchase in order to satisfy product specifications and product demands at minimum cost.

3.1.5 Production Distribution Problem: These problems occur when the products needed by the various destinations in a transportation problem do not list in finished form but rather must be manufactured at the sources before shipment. The sources may have different production costs. The problem then is to minimize cost by deciding what is to be produced at each source and where the goods are to be shipped.

3.1.6 Trim Loss: In many situations, products are made in standard sizes, while orders are received for materials in various shapes, sizes and quantities. The problem is to determine which combination of requirements should be produced from standard materials in order to keep trim loss to a minimum.

4. Applications in Management Area

4.1 Management Applications.

4.1.1 Portfolio Selection: A problem frequently encountered by managers of banks, insurance companies, investment services, credit unions is the selection of specific investments from among a wide variety of alternatives. The objective is minimization of risk. The constraints usually take the form of restrictions on the type of permissible investments, state laws, company policy, maximum permissible risk etc.

4.1.2 Financial Mix Strategy: These involve the selection of means for financial company projects, production operations and various other activities. Use of Linear Programming make decision making with regard o how much production is to supported by internally generated funds and how much to be supported to external funds easy.

4.1.3 Profit Planning: Here objective is to maximize profit margin from net investment in plant facilities and equipment, cash on hand, and inventory.

4.1.4 Media Selection: Linear Programming models are used in advertising field as a decision aid in selecting an effective media mix. They aimed at helping marketing managers in allocating a fixed or limited budget across various potential advertising media like newspapers, magazines,

ISSN 0976-9714

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radio and television commercials, direct mailings, etc. In most of these cases the objective is maximization of audience exposure. Restrictions on the allowable media mix might arise through contract requirements, limited media availability or company policy.

4.1.5 Travelling Salesmen Problem: The problem of finding the shortest route for a salesman starting from a given city, visiting each of the specified cities and then returning to the original point of departure can be easily handled by the assignment technique of linear programming.

4.1.6 Staffing problem: Development of work schedule that allows a large restaurant, hospital or police station to meet staff needs at all hours of the day. Minimizing the total number of employees is done by using Linear programming.

5. Applications in Other Areas

5.1 Miscellaneous Applications:

5.1.1 Farm planning: In farm management, there arises a typical problem which is concerned with the allocation of limited resources such as acreage, labour, water supply, and working capital in such a way that net revenue to be maximized. The problem is to choose simultaneously i) the particular crops to be grown during a period, ii) the acreage to be devoted each and iii)the particular production methods to be used.

5.1.2 Airline routine: Due to limited resources, Linear Programming techniques are used by a number of airlines to determine the most economic pattern and timing for flights, with the view to making the most efficient use of aircraft, crew, and money.

5.1.3 Administration: For optimal city administration, i.e. optimal usages of resources like men, machine, and material. Linear Programming helps an administrative incharge in knowing the best possible manner to utilize his resources. Linear Programming can be used to make decisions for departmental staff requirements for a given period of time. It can also be used to evaluate long-range hiring, promotion, and retirement schemes as well as proposed compensation schemes. LP techniques can also be used for work distribution among staff members according to their efficiency, so as to obtain optimum result.

5.1.4 Education: For allocating resources in education, for making school assignments in large districts etc.

5.1.5 Politics: In planning political campaign strategies, for resource allocations in local election campaigns.

ISSN 0976-9714

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5.1.6 Diet Problems: It involves specifying a food or feed ingradient combination that will satisfy stated nutritional requirements at a minimum cost level. These types of problems are used in hospitals to determine the most economical diet plan for patients. In Agricultural field this problem is known as feed mix problem.

5.1.7 Agriculture: Linear Programming techniques may be useful for land allocation and animal diet problems. LP may also help in determining the allocation of land for different crops/vegetables so that a farmer can either minimize his costs or maximizes profits subject to certain crop requirements.

5.1.8 Environmental Protection: To analyze alternatives for handling liquid waste material in order to satisfy antipollution requirements, LP has been used. It can also be used as a tool in the analysis of paper recycling, air cleaner design, and analysis of alternative energy policies.

5.1.9Urban Department: In this area LP has been used to analyze public expenditure planning, school busing, and drug control. Political redistricting and financial analysis of new community development processes have also been tackled with Linear Programming techniques.

5.2. Facilities Location: Linear Programming techniques are used in Location analysis i.e. location of public parks and recreation areas within communities, location of hospitals, ambulance depots, nuclear power plants, telephone exchanges, and warehouses for centralized distribution.

5.2.1 Cargo Loading Problem, Capital Budgeting Problem, Manpower Planning Problem,

5.2.2 Production Planning, Feed Mix, Stock Cutting or Slitting, Water-Quality Management,

Oil Drilling And Production, Assembly Balancing, Inventory. Determination of Equitable Salaries.

5.2.3 Some more areas where **Linear Programming** can be used are as follows.

*In structural design for maximum product.

*In scheduling of a military tanker fleet.

* In determining which parts to make and which to buy to obtain maximum profit margin.

*In selecting equipment and evaluating method improvements that maximize profit margin.

*In planning most profitable match of sales requirements to plant capacity that obtains a fair share of the market.

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*In design of optimal purchasing policies.

*Choice of investment from a variety of shares and debentures so as to maximize return on investment.

*Al location of a limited publicity budget on various heads in order to maximize its effectiveness.

*To determine the best way to obtain A variety of small rolls of smaller rolls of paper from a standard width of roll that is kept in stock and at the same time minimize wastage.

The location of Rajendra Bridge over Ganges linking South Bihar with North Bihar at Mokamah in preference to other sites has been done with the help of Linear Programming.

6. Non-Linear Programming

Nonlinear Programming is an extension of linear programming. In many real-life problems, the objective function may be nonlinear but the set of constraints may be linear or nonlinear.

6.1 Applications of Non-Linear Programming

6.1.1 Petrochemical Industry: Product blending, Refinery unit optimization, The unit design to multiplant production, Distribution planning.

6.1.2 Nonlinear network: Electric power dispatch, hydroelectric reservoir management, Problems involving traffic flow in urban transportation networks, Routing.

6.1.3 Economic Planning: Dynamic econometric models, Static equilibrium models, Submodels of larger planning system.

6.1.4 Miscellaneous:

Resource allocation, Computer aided design, Solution of equilibrium models, Data analysis & least square formulations, modeling human or organizational behavior.

7. Case Studies:

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7.1 (**Production Scheduling Problem)** A Company manufactures two types of articles A and B. The contribution for each article as calculated by the accounting department is Rs. 50 per A and Rs. 70 per B. Both products are processed on three machines M1 M2 and M3. The time required (in hours) by each article and the total time available per week on each machine are as follows.

Machine	A	B	Available hours per week
M1	3	2	46
M2	5	2	50
M3	2	6	70

How should the manufacturer schedule his production in order to maximize contribution? Formulate the problem as a LPP.

Solution: Steps: a). Key decision: To determine the number of units of article A and article B to maximize the profit while satisfying the conditions of hours of 3 machines.

b) Identify the decision variables and assign them symbols.

Let X_1 = number of units of article 'A'.

X_2 = number of units of article 'B'.

c) State the feasible alternatives i.e. Non negativity constraints.

Since X_1, X_2 denote number of units, it cannot be negative.

i.e. $X_1 \geq 0, X_2 \geq 0$.

d) Express the time constraints in terms of variables.

No. of hours required for X_1 units of type A article on machine **M1** = $3X_1$.

No. of hours required for X_2 units of type B article on machine **M1** = $2X_2$

. Maximum available hours per week on machine M1 = 46

Thus $3X_1 + 2X_2 \leq 46$. (constraint on M1)

Similarly $5X_1 + 2X_2 \leq 50$. (constraint on M2)

$2X_1 + 6X_2 \leq 70$. (constraint on M3)

e) Construct the Objective function.

Objective is Maximization of contribution i.e. profit.

Profit on X_1 units of 'A' = $50X_1$.

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Profit on X_2 units of 'B' = $70X_2$.

Total Profit $Z = 50X_1 + 70X_2$

Thus LPP model is as follows:

Maximize $Z = 50X_1 + 70X_2$.

Subject to $3X_1 + 2X_2 \leq 46.$

$5X_1 + 2X_2 \leq 50.$

$2X_1 + 6X_2 \leq 70.$

And $X_1 \geq 0, X_2 \geq 0.$

7.2 Manpower scheduling problem:

In a multi-specialty hospital, nurses report to duty at the end of every 4 hour as shown in the table given below. Each nurse, after reporting, will work for 8 hours continuously. The minimum numbers of nurses required during various periods are summarized in table below. Develop a mathematical model to determine the number of nurses to report at the beginning of each period such that the total number of nurses who have to report to duty in a day is minimized.

Interval No.	Time period		Minimum number of nurses required.
	From	To	
1	12 midnight	4 a.m.	20
2	4 a.m.	8 a.m.	25
3	8 a.m.	12 noon	35
4	12 noon	4 p.m.	32
5	4 p.m.	8 p.m.	22
6	8 p.m.	12 midnight	15

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Solution: Let X_i be the number of nurses to report for duty at the beginning of the i^{th} interval. $i = 1,2,\ldots\ldots.6$

Objective function : Minimize $Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

Subject to $X_6 + X_1 \geq 20$, $X_1 + X_2 \geq 25$, $X_2 + X_3 \geq 35$, $X_3 + X_4 \geq 32$,

$X_4 + X_5 \geq 22$, $X_5 + X_6 \geq 15$ **and**

$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$ **and integers.**

7.3.Cargo loading problem

Consider a problem where five items are to be loaded on a vessel. The weight(W) and volume (V) of each unit of the different items as well as their corresponding returns per unit (r_i) are tabulated as follows. The maximum cargo weight and volume are given as 112 and 109 respectively. It is required to determine the optimal cargo load in discrete units of each item such that the total return is maximized.

Item i	w_i	v_i	r_i
1	5	1	4
2	8	8	7
3	3	6	6
4	2	5	5
	7	4	4

Solution: Let X_i be the number of units of the i^{th} item to be loaded in the cargo where $i = 1,2,\ldots.5$.

Objective function: Return to be maximized:

$$\textbf{Maximize } Z = 4X_1 + 7X_2 + 6X_3 + 5X_4 + 4X_5$$

Subject to $5X_1 + 8X_2 + 3X_3 + 2X_4 + 7X_5 \leq 112$

$X_1 + 8X_2 + 6X_3 + 5X_4 + 4X_5 \leq 109$

$X_1, X_2, X_3, X_4, X_5 \geq 0$ **and integers.**

7.4 A company wants to engage casual labours to assemble its products daily. The company works for only one shift which consists of 8 hours and 6 days a week. The casual labours consist of two categories, viz. skilled and semi-skilled. The daily

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production per skilled labour is 80 assemblies and that of the semi skilled labour is 60 assemblies. The rejection rate of the assemblies produced by the skilled labours is 5% and that of semi skilled labours is 10%. The loss to the company for rejecting an assembly is Rs. 25. The daily wage per labour of the skilled and semi skilled are Rs. 240 and Rs. 160 respectively. The required weekly production is 1, 86,000 assemblies. The company wants to limit the number of semi-skilled labours per day to utmost 400. Determine the optimal mix of casual labours to be employed so that the total cost (total wage + total cost of rejections) is minimized.

Solution: Let X_1 be the number of skilled labours to be employed per day, X_2 be the number of semi skilled labours to be employed per day.

Total Wages: $240\ X_1 + 160\ X_2$

Total cost of rejection: $[4(5\%\ \text{of}\ 80)\ X25 =]100\ X_1 +$

$[6(10\%\ \text{of}\ 60)X25=]\ 150\ X_2$

i.e. $100X_1 + 150X_2$

Objective function: total Wages +total cost of rejection

$Z = 340X_1 + 310X_2$ to be minimized.

Subject to constraint on production:

Total minimum required production

per day = Weekly production/No. of days in week

= 186000/6 = 31000

From skilled labours out of 80 , 4 are rejected so 76 are acceptable

and from semi skilled out of 60 , 6 are rejected so 54 are acceptable.

Total no. of acceptable assemblies: $76X_1 + 54X_2$

Constraint on no. of semi skilled labours: utmost 400 i.e. $X_2 <= 400$

Thus LPP Model is **Minimize $Z = 340X_1 + 310X_2$**

Subject to $76X_1 + 54X_2 >= 31000$

ISSN 0976-9714

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$$\mathbf{X_2 <= 400}$$

$$\textbf{and} \qquad \mathbf{X_1, X_2 >= 0}$$

7.5. The MNS Company has been a producer of picture tubes for television sets and certain printed circuits for radios. The company has just expanded into full scale production and marketing of AM and AM-FM radios. It has built a new plant that can operate 48 hours per week. Production of an AM radio in the new plant will require 2 hours and production of an AM-FM radio will require 3 hours. Each AM radio will contribute Rs. 40 to profits while an AM-FM radio will contribute Rs. 80 to profits. The marketing department, after extensive research has determined that a maximum of 15 AM radios and 10 AM-FM radios can be sold each week. Determine the optimum production mix of AM-FM radios that will maximize profits.

Solution: Let X_1 denote number of AM radios, and X_2 denote number of AM-FM radios produced.

Since X_1 , X_2 denote number of units, it can not be negative. So

$$X_1, X_2 \geq 0$$

Time constraint on new plant: No. of hours required for X_1 AM radios are $2X_1$. No. of hours required for X_2 AM-FM radios are $3X_2$.

Total **Maximum** hours available for new plant = 48

Thus $2X1 + 3X_2 \leq 48$.

Objective function: Profit function: $Z = 40X_1 + 80X_2$ which is to be maximized.

LPP model is **Maximize $\mathbf{Z = 40X_1 + 80X_2}$**

Subject to $\mathbf{2X_1 + 3X_2 \leq 48}$

$$\mathbf{X_1 \leq 15}$$

$$\mathbf{X_2 \leq 10}$$

$$\mathbf{X_1, X_2 \geq 0}$$

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7.6 A Diet Problem: A diet conscious housewife wishes to ensure certain minimum intake of vitamins A, B, C for the family. The minimum daily needs of the vitamins A, B, C for the family are respectively 30, 20 and 16 units. For the supply of these minimum vitamin requirements, the housewife relies on two fresh foods. F1 and F2 . The F1 provides 7,5,2 units of the three vitamins per gram respectively and the F2 provides 2,4, 8 units of the same three vitamins per gram of the foodstuff respectively. F1 costs Rs. 3 per gram and F2 costs Rs. 2 per gram. How many grams of each foodstuff should the housewife buy everyday to keep her food bill as low as possible?

Solution: Let X gram of F1 and Y gram of F2 the housewife buy everyday.

Naturally $X \geq 0$ and $Y \geq 0$

The Objective function is Cost function (bill) which is to be minimized.

X units of F1 costs 3X and Y units of F2 cost 2Y Rs. Respectively.

Total cost $Z = 3X + 2Y$

For vitamin constraints the information given is tabulated as follows.

Food	Content of vitamins type A	Content of vitamins type B	Content of vitamins type C
X	7	5	2
Y	2	4	8
Minimum vitamins required	30	20	16

The Vitamin A constraint: $7X + 2Y \geq 30$

The Vitamin B constraint: $5X + 4Y \geq 20$

The Vitamin C constraint: $2X + 8Y \geq 16$

Thus LPP is Minimize Z = $3X + 2Y$ **Subject to** $7X + 2Y \geq 30$

$5X + 4Y \geq 20$

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$$2X + 8Y \geq 16$$

$$X \geq 0 \text{ and } Y \geq 0$$

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